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QUICK REVISION

FORMULA SHEET

for

GATE-AE SPACE DYNAMICS

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AIR - 7

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SPACE DYNAMICS

GRAVITATIONAL BASICS:

Constants:

1. $G = 6.67 \times 10^{-11} Nm^2/kg^2$
2. $Re = 6371$ km
3. $M_e = 5.98 \times 10^{24}$ kg
4. $\mu = GM_e = 3.98 \times 10^{14}\ m^3/s^2$
5. $g_o = 9.81\ m/s^2$

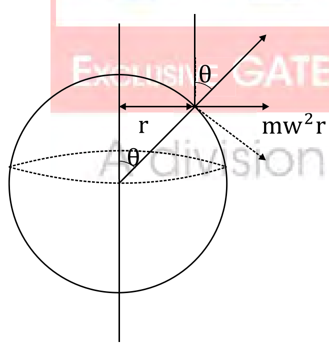
Variation of 'g' from Earth's surface:

$g = g_o\left[1 - \frac{2h}{R}\right]$ above earth

$g = g_o\left[1 - \frac{h}{R}\right]$ below earth

$g_o = \frac{GM}{r^2}$

Effect of Earth's Rotation:



$g' = g - \omega r \sin\theta$

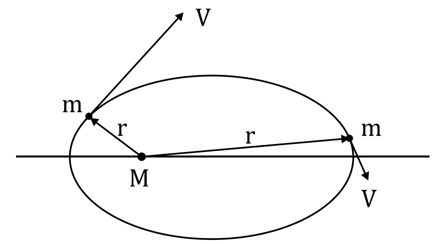
Note: $g \to$ max at poles $(\theta = 0°)$

$g \to$ min at equator $(\theta = 90°)$

KEPLER'S LAW:

First law

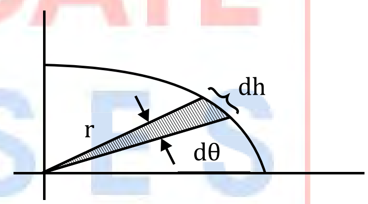
A satellite or object describes an elliptical path around its centre of attraction.



Its path depends on velocity.

Second law

The area swept out by radius vector of object in equal time remains same.



$\frac{dA}{dt} = \text{constant}$

Third law

The square of time period is proportional to cube of semi-major axis.

$T^2 \propto a^3$

ORBITAL TRAJECTORY:

Lagrange's Function:

$\beta = T - \phi \quad T \to +Ve, \phi \to -Ve$

Eventually sums up

$T \to k.E, \phi = P.E$

Lagrange's equation of motion

$\frac{d}{dt}\left(\frac{\partial\beta}{\partial q_1}\right) - \left(\frac{\partial\beta}{\partial q_1}\right) = 0\,; \quad q_1 \to \text{Coordinate}$

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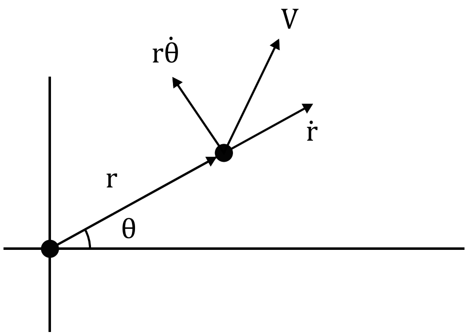
Orbital Equation:



$$T = \frac{1}{2} mV^2$$

$$V^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$\text{and } \phi = -\frac{GM_m}{r}$$

$$\therefore \beta = T - \phi = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu m}{r}$$

From Lagrange equation of motion.

For θ Coordinate:

We get $r^2\dot{\theta} = \frac{\text{const}}{m} = h$

Angular momentum per unit mass.

For r-Coordinate:

We get equation of trajectory of an object under conservative force

$$r = \frac{h^2/\mu}{1 + e\cos(\theta - c)}$$

Here $e =$ eccentricity.

Orbit Types:

$e = 0$, circular

$e < 1$, Elliptiocal

$e = 1$, Parabolic

$e > 1$, Hyperbolic

Orbital Energy:

$$\text{k.E} + \text{P.E} = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - \frac{\mu m}{r}$$

$$\text{Per unit mass, } E_T = \frac{V^2}{2} - \frac{\mu}{r} = \text{const}$$

$$\left[V^2 = \dot{r}^2 + r^2\dot{\theta}^2\right]$$

Relation Between e and E_T:

$$e = \sqrt{1 + \frac{2h^2}{\mu^2} E_T}$$

Cases:

a. $e < 1$, Then $E_T < 0 \Rightarrow \left|\frac{-\mu}{r}\right| > \left|\frac{V^2}{2}\right|$

Elliptical Orbit

b. $e = 1$, then $E_T = 0 \Rightarrow \left|\frac{-\mu}{r}\right| = \left|\frac{V^2}{2}\right|$

Parabolic Trajectory

c. $e > 1$, then $E_T > 0 \Rightarrow \left|\frac{-\mu}{r}\right| < \left|\frac{V^2}{2}\right|$

Hyperbolic Trajectory

Escape Velocity:

To escape earth's gravitation objects kinetic energy, need to be equal to potential energy

$$E_T = 0 \Rightarrow \frac{V^2}{2} - \frac{\mu}{r} = 0 \Rightarrow V_e = \sqrt{\frac{2\mu}{r}}$$

$$V_e = \sqrt{\frac{2GM}{(R_e)}} \qquad r = R_e + h$$

(or)

$$V_e = \sqrt{2g_o(R_e)}$$

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ORBITS:

1. **Circular:**

$e = 0, E_t < 0$

$$E_T = \frac{-\mu^2}{2h^2}\left[\therefore e = \sqrt{1 + \frac{2h^2}{\mu^2}ET}\right]$$

Also $E_T = \frac{V^2}{2} - \frac{\mu}{r}$

$$\frac{-\mu^2}{2h^2} = \frac{V^2}{2} - \frac{\mu}{r}$$

For Circular orbit r= h^2/μ

So

$\therefore$ Velocity for circular orbit, $V_o = \sqrt{\frac{\mu}{r}} = \frac{\mu}{h}$

Escape Velocity from Circular Orbit:

$$\Delta V_{escape} = \sqrt{\frac{2\mu}{r}} - \sqrt{\frac{\mu}{r}}$$

Time Period:

$$T^2 = \frac{4\pi^2}{GM}, R^3$$

2. **Parabolic:** $e = 1,\ E_T = 0$

Velocity for parabolic orbit,

$V = \sqrt{\frac{2\mu}{r}}$ (same as escape velocity)

$\rightarrow r_p = \frac{h^2}{2\mu} \rightarrow$ At perigee, $\theta = 0°$

$\rightarrow$ Velocity at pergee, $V_{max} = \sqrt{\frac{2\mu}{r_p}} = \frac{2\mu}{h}$

3. **Hyperbolic:** $e > 1, E_T > 0$

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Radius at Perigee, $r_p = \frac{h^2/\mu}{1+e} = a(e-1)$

Orbital Energy, $E_T = \frac{\mu}{2a}$

Velocity for hyperbolic orbit

$$V = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} + 1\right)}$$

Velocity at perigee,

$$V_P = \sqrt{\frac{\mu}{a}\left(\frac{e+1}{e-1}\right)}$$

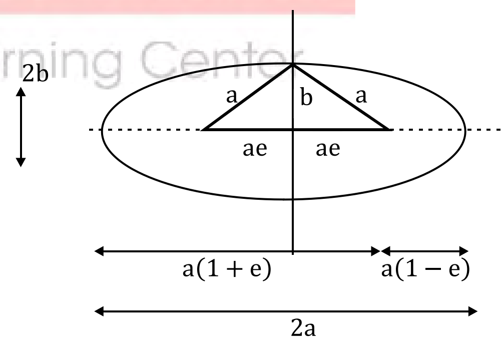
4. **Elliptical Orbit:**

a= Semi major axis

b= Semi minor axis

For ellipse

$r_p + r_a = 2a;\ \ r_a - r_p = 2ae$



Radius at Apogee, $r_a = a(1+e)$

Perigee, $r_p = a(1-e)$

Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$e = \frac{r_a - r_p}{r_a + r_p}$$

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Energy at orbit, $E_T = \frac{-\mu}{2a}$

Velocity at orbit, $V = \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} - 1\right)}$

Velocity at perigee, $V_p = \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)}$

or

$$V_p = \sqrt{\frac{2\mu r_a}{r_p(r_a + r_p)}}$$

Velocity at apogee, $V_a = \sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)}$

or

$$V_a = \sqrt{\frac{2\mu r_p}{r_a(r_a + r_p)}}$$

→ Ratio of Velocities, $\frac{V_p}{V_a} = \frac{(1+e)}{(1-e)}$

→ Escape from elliptical orbit

$$\Delta V_{esc} = \sqrt{\frac{2\mu}{r}} - \sqrt{\frac{\mu}{a}\left(\frac{2a}{r} - 1\right)}$$

→ Time Period $T^2 = \frac{4\pi^2}{\mu} a^3$

ADDITIONAL CONCEPTS

Orbital Maneuvers

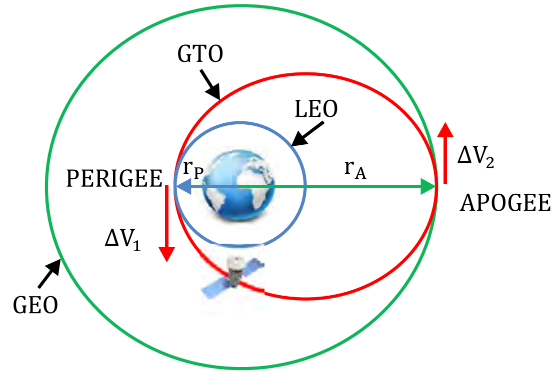
(1) Earth-satellite system

(a) Hohmann transfer

(b) Inclination Change maneuver

Earth – Satellite system

1. **Hohmann Transfer** for Raising Satellite orbit from circular LEO to GEO



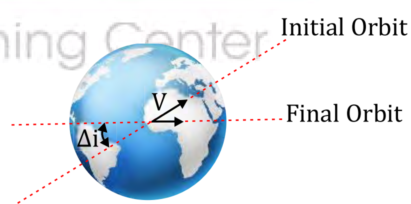
The total ΔV required for the Hohmann transfer will then be

$\Delta V_{total} = \Delta V_1 + \Delta V_2$

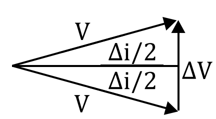
$$= \left(\sqrt{\frac{2\mu r_A}{r_P(r_P + r_A)}} - \sqrt{\frac{\mu}{r_P}}\right) + \left(\sqrt{\frac{\mu}{r_A}} - \sqrt{\frac{2\mu r_A}{r_A(r_P + r_A)}}\right)$$

2. **Inclination Change Maneuver**

Simple Inclination change: Maneuver must occur where the two orbits intersect.



If 'V' be the circular velocity of the spacecraft and 'Δi' be the inclination change required.



$$\Delta V = 2V \sin\left(\frac{\Delta i}{2}\right)$$

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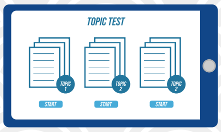

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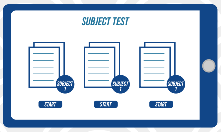

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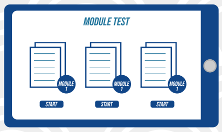

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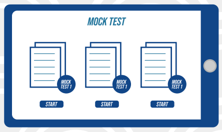

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